

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Number

Problem solving: Finding the HCF and LCM

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

(a) 1993

B1

(b) 2021

B1

(c) Valid reason

*eg**2009 + multiple of 4 can never be a multiple of 4**Always 1 year after a leap year**Always in an odd year***B1****Additional Guidance**

09 is not a leap year and every 4 years

B1

09 is not divisible by 4

B0

Always between leap years

B0**[3]****Q2.**

(a) 2010

B1

(b) 2046

B1

(c) Valid reason

*eg**(20)22 + multiple of 8 can never be a multiple of 4**Always 2 years before (or after) a leap year***B1****Additional Guidance**

2022 is not a leap year and every 8 years

B1

22 is not divisible by 4

B0

Always between leap years

B0**[3]**

Q3.

Alternative method 1

Lists the multiples of two of 10, 8, 5

10, 20, 30, 40, ...

8, 16, 24, 32, 40, ...

5, 10, 15, ..., 40, ...

Writes out all the multiples to at least 40

M1

40

May be implied by correct number of boxes

A1

4

and 5

and 8

ft their multiple of 40

B1ft

Alternative method 2

Lists the prime factors of two of

10, 8, 5

$10 = 2 \times 5$

$8 = 2 \times 2 \times 2$

$5 = 5 (\times 1)$

M1

$2 \times 2 \times 2 \times 5$

May be implied by correct number of boxes

A1

4

and 5

and 8

ft their multiple of 40

B1ft

[3]

Q4.

(a) $4xy$

B1

(b) $8x^2y^3$

B1

[2]

Q5.

Alternative method 1

Correctly lists first three bus times to X or Y

ie

7 25, 7 50, 8 15, ...

or

7 20, 7 40, 8 00, ...

Accept any notation for time eg 7.20, 7:20

7 20, 0720, 7-20, 20 past 7, 720

M1

Continues **both** lists at least as far as a common time

ie

7 25, 7 50, 8 15, 8 40, ...

and

7 20, 7 40, 8 00, 8 20, 8 40, ...

Allow one error up to and including their common time, ignore errors after.

M1dep

8.40 (am) or 08 40 or after/in 100 minutes or after/in 1h 40 minutes

SC2 No other working and any time that is 7 am + 100n minutes, eg 10 20, 12 00, 13 40 etc..

A1

Alternative method 2

Correctly lists first three multiples of 25 or 20

ie

25, 50, 75, ...

or

20, 40, 60, ...

25×4 and 20×5

M1

Stops both lists at 100 or identifies 100 or 1 hour 40 minutes

M1dep

8.40 (am) or 08 40 or after/in 100 minutes or after/in 1h 40 minutes

SC2 No other working and any time that is 7 am + 100n minutes, eg 10 20, 12 00, 13 40 etc..

A1

Additional Guidance

7 25, 7 50, 8 15, 8 40, 9 05, ...

M1

7 20, 7 40, 8 00, 8 20, 8 40, 9 00, ..

M1dep

(Answer =) 8 40 pm

pm is wrong.

A0

(No working)

(Answer =) 8 40 pm

Method by implication

M2

7 25, 7 50, 8 05, 8 30, 8 55, 9 20

Second list correct for 3 values.

M1

7 20, 7 40, 8 00, 8 20, 8 40, 9 00 9 20 One error in first list.

M1dep

(Answer =) 9 20

Both lists taken to a common value.

A0

7 25, 7 50, 8 10, 8 30, 9 00, 9 15

Second list correct for 3 values.

M1

7 20, 7 40, 8 00, 8 20, 8 40, 9 00

Both lists taken to a common value but more than one error in first list.

M0dep

(Answer =) 9 00

A0

25, 50, 75, 80, *At least one list correct for 3 values.*

M1

20, 40, 60, 80, *Does not get to 100*

M0

(Answer =) 8 10

A0

7 00, 25, 50, 8 15, 40, 9 05, ...

M1

7 00, 20, 40, 8 00, 20, 40, 9 00 *Intention to list times clear*

M1dep

8 40

A1

As question asks for 'When..' rather than 'What time..' then the students do not have to say 8.40 but could qualify it as a length of time **after** 7am. If so then the wording must be clear.

7 25, 7 50, 8 15, 8 40, 9 05, ...

M1

7 20, 7 40, 8 00, 8 20, 8 40, 9 00, ..

Must make it clear that the time is after 7 (am)

M1dep

(Answer =) 1 h 40 after 7

A1

7 25, 7 50, 8 15, 8 40, 9 05, ...

M1,

7 20, 7 40, 8 00, 8 20, 8 40, 9 00, .. *Not clear that the time is after 7 am*

M1dep

(Answer =) 1 h 40

A0

[3]

Q6.

(a) $8x^4y^7$

B1 for two out of three parts correct eg $6x^4y^7$

B2

(b) $4y(5y - 2x)$

*B1 for $4y(? - ?)$
or $4(5y^2 - 2xy)$ or $8y(2.5y - x)$ or $y(20y - 8x)$
or $8(2.5y^2 - xy)$ or $2(10y^2 - 4xy)$ or $2y(10y - 4x)$*

B2

(c) $w - y = \frac{x}{r}$

$wr = yr + x$ or $-x = yr - wr$ oe

M1

$r(w - y) = x$

$wr - yr = x$

Must have $x = \dots$ oe

A1

(d) $6x^2y^2$

B1 for $18x^3y^3$ or any other common multiple

B2

[8]

Q7.

$2xy$

B1

[1]

Q8.

(a) $200 \div 10 (= 20)$

or

$20 \times 10 = 200$

or

$200 \div 20 = 10$

B1

(b) (Becky) (25 50 75) 100 **and**

(Chris) (20 40 60 80) 100

100 (or multiple of 100) as a common multiple

B1

4×200

ft 4 from their lowest common multiple

4 must be from Becky's multiples

M1

800

ft B0 M1

SC1 1000

SC1 Any multiple of 800

A1ft

Alternative 1

(Becky) (8 16 24 32) 40 **and**

(Chris) (10 20 30) 40

40 (or multiple of 40) as a common multiple

B1

4×200

ft 4 from their lowest common multiple

4 must be from Chris's multiples

M1

800

ft B0 M1

SC1 1000

SC1 Any multiple of 800

A1ft

Alternative 2

$200 \div 2 (= 100)$

2 is the difference in speeds

M1

their 100×8

M1

800

SC1 1000

SC1 Any multiple of 800

A1

[4]

Q9.

4 packs of bread rolls and 25 packs of sausages

B2 4n packs of bread rolls and

25n packs of sausages

where n is an integer > 2

e.g. 8 packs of bread rolls and 50 packs of sausages

B1 Works out a common multiple of 8 and 25

e.g.1 8, 80, 160, 200 and 25, 50, 100, 200, 250

e.g.2 $8 \times 25 = 200$

e.g.3 $2^3 \times 5^2 = 200$

or Indicates a valid number of bread rolls and sausages

i.e. 100m bread rolls and

200m sausages

where m is an integer > 0

SC2 25 packs of bread rolls and 4 packs of sausages

B3

[3]

Q10.

(a) $2^3 \times 3 \times a^2$ or $24a^2$ (= 4056)

or

4056

$a = 2^3 \times 3$ or ($a^2 =$) 169

or

$\sqrt{169}$ oe eg $8 \times 3 \times a^2$

M1

13

A1

Additional Guidance

Condone $a^2 \times 24$ for M1

Fully correct prime factor decomposition with values 2, 2, 2, 3, 13, 13 shown without 13 chosen as the final answer

M1A0

Embedded answer $2^3 \times 3 \times 13^2$

M1A0

± 13 or -13

M1A0

$4056 \div 2^3 \times 3$ unless recovered to 169

M0A0

(b) $2^4 \times 3^2 \times a^3$ or $144a^3$

or $2^4 \times 3^2 \times (\text{their } 13)^3$

or $13 \times 4056 \times 2 \times 3$

or 52728×6

or 24336×13 oe eg $144 \times (\text{their } 13)^3$

$16 \times 9 \times 2197$

M1

316 368 ft their 13, which must be an integer > 13

A1ft

Additional Guidance

eg 14 on answer line in part (a) can follow through to $144 \times 14^3 = 395136$

M1A1ft

[4]

Q11.

Alternative method 1

Valid number of bread rolls and cheese slices

e.g. 30 bread and 60 cheese

or 60 bread and 120 cheese

or 90 bread and 180 cheese

or 120 bread and 240 cheese

Valid number means ratio 1 : 2 and can be bought in exact numbers of packs

May be implied by valid number of packs

M1

Valid number of packs of bread rolls and cheese slices

e.g. 2 packs bread and 3 packs cheese

or 4 packs bread and 6 packs cheese

or 6 packs bread and 9 packs cheese

or 8 packs bread and 12 packs cheese

Valid number of packs means ratio 2 : 3

M1dep

Their number of packs of bread $\times 1.88$

and their number of packs of cheese $\times 2.15$

e.g. 15.04 and 25.8(0)

M1dep

40.84

SC2 27.94 or 42.98

A1

Alternative method 2

Valid number of sandwiches

e.g. Common multiple of 15 and 20 identified

e.g. 15 30 45 60 75

and 20 40 60

Valid number means can be bought in exact numbers of packs

M1

$1.88 \div 15 + 2.15 \div 10$ or $0.125(\dots) + 0.215$ or $0.340(\dots)$

Oe Cost of one sandwich

M1

their $0.340(\dots) \times$ their number of sandwiches

dep on M2

M1dep

40.84

SC2 27.94 or 42.98

A1

Additional Guidance

Alt 1 3rd M1 Allow working in pence

Alt 2 2nd M1 Allow working in pence

30 bread and 60 cheese/2 packs bread and 3 packs cheese

2×1.88 or 3.76 **and** 3×2.15 or 6.45 (Answer £10.21)

M3 A0

60 bread and 120 cheese/4 packs bread and 6 packs cheese

4×1.88 or 7.52 **and** 6×2.15 or 12.9(0) (Answer £20.42)

M3 A0

90 bread and 180 cheese/6 packs bread and 9 packs cheese

6×1.88 or 11.28 **and** 9×2.15 or 19.35 (Answer £30.63)

M3 A0

150 bread and 300 cheese/10 packs bread and 15 packs cheese

10×1.88 or 18.8(0) **and** 15×2.15 or 32.25 (Answer £51.05)

M3 A0

SC2 from 120 bread and 120 cheese or 240 bread and 120 cheese

[4]

Q12.

- (a) Pair of values of form $2x$ and kx
where x is an integer > 5
and k is an odd integer > 2
eg 12 and 18 or 100 and 250

B1

- (b) Ticks No with valid reason including that one number could be 2 and that multiplying by an even number (or 2) gives an even answer

*eg1 Ticks No and a could be 2
and the others will be odd*

and even $\times$ odd $\times$ odd = even

*eg2 Ticks No (and b could be 2)
and $27 \times 4 \times 5 = 540$*

eg3 Ticks No and $125 \times 9 \times 2 = 2250$

*eg4 Ticks No and a, b or c could be 2 and
multiplying by an even gives an even*

B1 No with partial reason

eg1 Ticks No and a could be even

eg2 Ticks No and $c = 2$

*eg3 Ticks No and odd $\times$ odd $\times$ even is
even*

*eg4 Ticks No and multiplying by an even
gives an even*

B2

Additional Guidance

If a box is not ticked, No can be implied by the reason for B2 and B1

eg1 a could be 2 and the others will be odd and even $\times$ odd $\times$ odd = even

B2

eg2 b could be 2 and $27 \times 4 \times 5 = 540$ which is even

B2

eg3 odd $\times$ odd $\times$ even is even

B1

Allow use of numbers that are not prime for B1

eg1 Ticks No and b could be 2 and $1 \times 4 \times 5 = 20$

B1

eg2 Ticks No and $4^3 \times 3^2 \times 2 = 1152$

B1

[3]

Q13.

- (a) 4 and 5
or 4 and 7
or 4 and 9

B1

- (b) Ticks No with valid reason

eg Ticks No, the smallest value of N is $2 \times 3 \times 5 = 30$.

or

Ticks No, you can only get $N = 6$ from $1 \times 2 \times 3$ and 1 is not a prime number.

B1 No with partial reason

eg Ticks No and it can't be 1.

B2

Additional Guidance

If a box is not ticked, No can be implied by the reason for B2 and B1

eg1 the smallest value of N is $2 \times 3 \times 5 = 30$.

B2

eg2 you can only get $N = 6$ from $1 \times 2 \times 3$ and 1 is not a prime number.

B2

[3]